Journal of Structural Geology, Vol. 15, No. 2, pp. 219 to 225, 1993
Printed in Great Britain

0191-8141/93 $06.00+0.00

The extremum rule of angular variation of finite deformation and its application in structural geology

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(*Received* 6 *January* 1992; *accepted in revised form* 12 *July* 1992)

Abstract—This paper is devoted to finite deformation theory and more specifically to the dispersion of cones of lines about an axis in the neighbourhood of a point. The paper has been fully verified that when a small angle $\Delta\theta$ in an initial sphere changes to $\Delta\theta$ in the strain ellipsoid, the limit ratio $\delta\theta/\delta\Theta$ has extrema, provided $\Delta\Theta$ is within the second principal plane and contains the first or third principal stretch axis. The extremum rule of angular variation is applied for principal stretch axes. For uniaxial tensile, pure shear, slide and simple shear, the author has derived the formulae of the principal stretch axes, and illustrated the equivalence relation of the rotation angle of the principal axes to the mean rotation angle of all line elements passing through a point. To determine the principal stretch axes of a fossil deformed in two dimensions, the author has used the curve of θ against Θ, i.e. the orientation angle of a mark line in the deformed fossil against the one in the undeformed fossil. In the last part of this paper the analytical expressions of extreme stretch trajectories in a heterogeneous shear zone with the distribution of parabolic displacement are obtained.

INTRODUCTION

LARGE deformation is an important phenomenon in geology. The fundamental theory of finite deformation is due to Cauchy (1823, 1827, 1841). Since then, the theory of finite deformation has advanced considerably (Truesdell & Toupin 1960, Truesdell & Noll 1965). Many geometric techniques and analytical approaches to structural deformation have been put forward (Breddin 1956, Flinn 1962, Wellman 1962, Ramsay 1967, Cobbold 1979, 1988, De Paor 1983, Ramsay & Huber 1983). In finite deformation, the dispersion of cones of lines about an axis is of great interest to geologists (March 1932, Owens 1973, De Paor 1981), because changes in angles as a result of strain are often used in determining the strain state. Based on finite deformation theory, this paper first verifies that the ratio of angular variation $\delta\theta/\delta\Theta$ possesses extrema. In the second section of the paper, the extremum rule is applied to determine the ratio of principal elongations and their orientations in a two-dimensional homogeneous deformation field as well as in a deformed fossil. The last part of this paper shows how to obtain the principal stretch trajectories in a heterogeneous shear zone where the displacement field has been shown.

THE EXTREMUM RULE OF ANGULAR VARIATION IN THE NEIGHBOURHOOD OF A POINT

Truesdell & Toupin (1960) comprehensively reviewed the theory of finite deformation and its historical development up to the early 1960s with abundant references. We summarize some important features of finite deformation: an infinitesimal material sphere at space point P becomes an ellipsoid at space point p; the principal axes of the ellipsoid are mutually perpendicular, both before and after deformation; if along these axes the stretches take extrema $\lambda_1^{1/2}$, $\lambda_2^{1/2}$ and $\lambda_3^{1/2}$ ($\lambda_1^{1/2} \geqslant \lambda_2^{1/2} \geqslant \lambda_3^{1/2}$), they stand in the same ratios as the lengths of the principal semi-axes of the ellipsoid; perpendicular diameters of the sphere at P become conjugate diameters of the ellipsoid at p. We now prove a new feature, the extremum rule of angular variation:

Let $\mathbf{N}$ be a unit vector along $\mathbf{dP}$, which is a small vector at point P (Fig. 1). An original small angle $\Delta\Theta$ between $\mathbf{dP}$ and $\mathbf{dP}'$, a small vector near $\mathbf{dP}$ at P, is deformed into a small angle $\Delta\theta$. If $\Delta\Theta$ is in the second principal plane, then the limit ratio $\delta\theta(\mathbf{N})/\delta\Theta$ maximizes to $(\lambda_1/\lambda_3)^{1/2}$ in the third principal direction (in which the stretch is minimal) and minimizes to $(\lambda_3/\lambda_1)^{1/2}$ in the first principal direction (in which the stretch is maximum). For arbitrary direction $\mathbf{N}$, there exists in general

$$\left(\frac{\lambda_3}{\lambda_1}\right)^{1/2} \leqslant \frac{\delta\theta(\mathbf{N})}{\delta\Theta} \leqslant \left(\frac{\lambda_1}{\lambda_3}\right)^{1/2}. \tag{1}$$

Proof

Verification will be conducted in the neighbourhood of a point. For the convenience of discussion, we let the radius of the original sphere be a unit and the lengths of principal semiaxes of the ellipsoid be $\lambda_1^{1/2}$, $\lambda_2^{1/2}$ and $\lambda_3^{1/2}$. In Fig. 1,

$$|\mathbf{dP}| = \mathrm{PQ} = |\mathbf{dP}'| = \mathrm{PQ}' = 1 \tag{2}$$

and the angle $\Delta\Theta$ between PQ and PQ′ is set in plane W. After deformation, $\mathbf{dP} \rightarrow \mathbf{dp} = \overrightarrow{\mathrm{pq}}$, $\mathbf{dP}' \rightarrow \mathbf{dp}' = \overrightarrow{\mathrm{pq}}'$, plane W → plane w, circle $L \rightarrow$ ellipse l, $\Delta\Theta \rightarrow \Delta\theta$. Geometrically let qs be perpendicular to pq′ and pk = pq. If $\Delta\Theta$ is small enough, then we have approximately

$$(\Delta\Theta)^2 \approx (\mathrm{QQ}')^2 \tag{3}$$

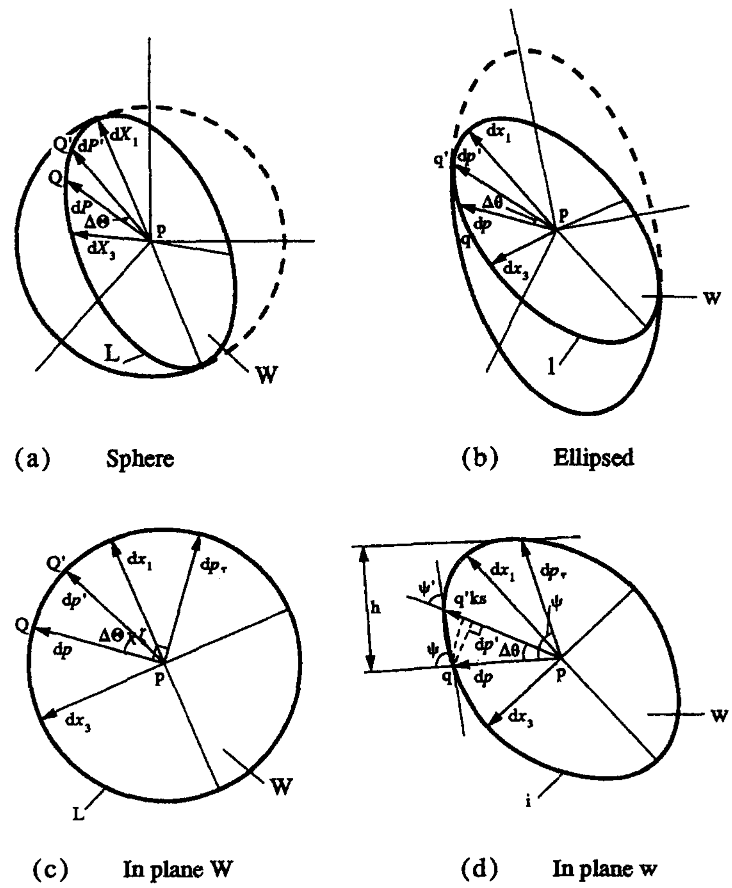

Fig. 1. Small-angle transformation in the neighbourhood of a point.

$$(\Delta\theta)^2 \approx (\mathrm{qk/pq})^2 \approx (\mathrm{qs/pq})^2 = (\mathrm{qq}' \cdot \sin\psi'/pq)^2 \quad (4)$$

$$\Delta\theta/\Delta\Theta \approx \mathrm{qq}' \cdot \sin\psi'/(\mathrm{QQ}' \cdot \mathrm{pq}). \quad (5)$$

Taking $\Delta\Theta \to 0$, then $\psi' \to \psi$, it follows that

$$\frac{\delta\theta}{\delta\Theta} = \frac{\sin\psi}{\mathrm{pq}} \lim_{\Delta\Theta\to 0} (\mathrm{qq}'/\mathrm{QQ}'), \quad (6)$$

where ψ is the acute angle between pq and the tangent to the ellipse l at point q. Evidently $\lim_{\Delta\Theta\to 0}(\mathrm{qq}'/\mathrm{QQ}')$ is the stretch along the tangent, namely the stretch in the direction of $\mathbf{dp}_\tau$, which conjugates with $\mathbf{dp}$ to the ellipse l, therefore

$$\lim_{\Delta\Theta\to 0} (\mathrm{qq}'/\mathrm{QQ}') = |\mathbf{dp}_\tau|/|\mathbf{dP}_\tau| = |\mathbf{dp}_\tau|, \quad (7)$$

here $\mathbf{dP}_\tau$ in the circle L is the inverse image of $\mathbf{dp}_\tau$. Consequently, equation (6) takes the simpler form

$$\delta\theta/\delta\Theta = |\mathbf{dp}_\tau| \cdot \sin\psi/|\mathbf{dp}| = \mathrm{h}/|\mathbf{dp}| \quad (8)$$

or

$$\begin{aligned}\delta\theta/\delta\Theta &= |\mathbf{dp}_\tau| \cdot \cos\gamma(\mathbf{N},\mathbf{N}_\tau)/|\mathbf{dp}| \\ &= (\lambda_\tau/\lambda)^{1/2}\cos\gamma(\mathbf{N},\mathbf{N}_\tau). \quad (9)\end{aligned}$$

In the above $\lambda_\tau^{1/2} = |\mathbf{dp}_\tau|$, $\lambda^{1/2} = |\mathbf{dp}|$, $\gamma(\mathbf{N},\mathbf{N}_\tau) = \pi/2 - \psi$, i.e. the shear of the directions $\mathbf{N} = \mathbf{dP}$ and $\mathbf{N}_\tau = \mathbf{dP}_\tau$. Writing $\Lambda_1^{1/2} = |\mathbf{dx}_1|$ and $\Lambda_3^{1/2} = |\mathbf{dx}_3|$ for the lengths of the major and minor semi-axes of ellipse l. If $\mathbf{dp} = \pm\mathbf{dx}_1$, then $\mathbf{dp}_\tau = \mathbf{dx}_3$, and $\gamma(\mathbf{N},\mathbf{N}_\tau) = 0$. From equation (8), we have

$$\delta\theta/\delta\Theta = (\Lambda_3/\Lambda_1)^{1/2}. \quad (10)$$

In reverse, if $\mathbf{dp} = \pm\mathbf{dx}_3$, then

$$\delta\theta/\delta\Theta = (\Lambda_1/\Lambda_3)^{1/2}. \quad (11)$$

But $\Lambda_3^{1/2} \leqslant h$, $|\mathbf{dp}| \leqslant \Lambda_1^{1/2}$, there exists generally

$$(\Lambda_3/\Lambda_1)^{1/2} \leqslant \delta\theta/\delta\Theta \leqslant (\Lambda_1/\Lambda_3)^{1/2}. \quad (12)$$

Nevertheless $\Lambda_1^{1/2} \leqslant \lambda_1^{1/2}$ and $\lambda_3^{1/2} \leqslant \Lambda_3^{1/2}$, it follows that

$$\left(\frac{\lambda_3}{\lambda_1}\right)^{1/2} \leqslant \frac{\delta\theta(\mathbf{N})}{\delta\Theta} \leqslant \left(\frac{\lambda_1}{\lambda_3}\right)^{1/2}. \quad (13)$$

This is valid for arbitrary direction $\mathbf{N}$ in the sphere. In equation (13), the left or right equality holds if and only if $\Delta\Theta$ is in the second principal plane and $\mathbf{N}$ coincides with the first principal direction $\mathbf{N}_1$ or the third principal direction $\mathbf{N}_3$. Hence the proof is completed in three dimensions.

If deformation is restricted to two dimensions, equation (13) is easily derived from the following formula (Ramsay 1967):

$$\tan\theta = \left(\frac{\lambda_3}{\lambda_1}\right)^{1/2}\tan\Theta. \quad (14)$$

For incompressible material, we have the equation

$$(\lambda_1\lambda_2\lambda_3)^{1/2} = 1. \tag{15}$$

If deformation is two-dimensional and $\lambda_2^{1/2} = 1$, we get the extremal stretches from equation (13) and (15) as follows:

$$\lambda_1 = \delta\theta(\mathrm{N}_3)/\delta\Theta, \qquad \lambda_3 = \delta\theta(\mathrm{N}_1)/\delta\Theta. \tag{16}$$

APPLYING THE EXTREMUM RULE TO DETERMINE THE PRINCIPAL STRETCH AXES OF TWO-DIMENSIONAL STRAIN

In case of large deformation, the principal directions and fibres of strain change with deformation. By means of the extremum rule of angular variation, we can find the major and minor axes of a strained ellipse on any microplane through a given point, provided the relation of $\Delta\theta$ to $\Delta\Theta$ in the plane would be known regardless of which direction is defined as the initial value of θ or Θ.

Table 1 gives some formulae for analysis of strain in two-dimensional deformation fields: uniaxial tensile, pure shear, slide and simple shear. Let $\phi = \theta - \Theta$ be the rotation angle of a line element with the initial azimuth Θ. As the extremum points of $\mathrm{d}\theta/\mathrm{d}\Theta$, principal directions Θ_1 and Θ_2 must respond to the inflection points of the ϕ–Θ curve. In Fig. 2, the principal directions Θ_1 and Θ_2, determined by the inflection points P and Q, are well consistent with the results from Table 1. Moreover the area under ϕ–Θ curve convinces us that the rotation angle of a principal stretch axis is equal to $\bar{\phi}$, the mean value of the rotation angles of all line elements passing through the point.

For uniaxial tensile or pure shear, we may well say that the neighbourhood of a point as a whole does not rotate or $\bar{\phi}$ is equal to zero, because the rotations of line elements are symmetric about the principal axes so that the area summed under the ϕ–Θ curve vanishes. In such cases, $\Theta_{1,2}$ and $\theta_{1,2}$ remain constant during deformation, therefore the principal directions and fibres are fixed all along.

For slide or simple shear, since the area summed under the curve is negative, the mean rotation angle $\bar{\phi}$ is negative too. We may consider that the neighbourhood of a point as a whole or each line element in the neighbourhood rotates clockwise at the angle $\bar{\phi}$. As the angle $\bar{\phi}$ is relevant to shear amount γ, the principal directions vary with γ. In case of simple shear, the fibre possessing the first principal direction will alternate from the original $\pi/4$ fibre to the $\pi/2$ fibre when γ is increasing. But in case of slide, the principal fibres do not vary with γ. Figure 2(c) also shows us two interesting phenomena which could not be predicted by small deformation theory: (1) some line elements rotate at larger angles than γ and (2) others rotate against γ.

In structural geology, deformed fossils are generally used as almost perfect strain gauges to determine the state of finite strain in sedimentary strata, if the fossils were originally interred in sediment and are strained homogeneously with the rock that encloses them. According to the extremum rule, we can determine the ratio of principal elongation $(\lambda_2/\lambda_1)^{1/2}$ and principal stretch axes within a single deformed fossil such as brachiopod, lamellibranch and so on by using the θ–Θ curve, where Θ (or θ) is defined as an angle in the undeformed (or deformed) state. Figure 3(a) shows a deformed brachiopod. We arbitrarily take two azimuths as the initial values of Θ and θ, and assume that the original lines OA_0–OA_8 were regularly scattered in the

Table 1. Formulae of strain analysis in two dimensions are derived by using the extremum rule of angular variation. $(X,Y)/(x,y)$, co-ordinates of a material point with respect to a fixed co-ordinate frame before and after deformation; Θ/θ, the angle included between a line element and horizontal axis before/after deformation; Θ_1/Θ_2, the first/second principal stretch direction before deformation; θ_1/θ_2, the first/second principal stretch direction after deformation; $\bar{\phi}$, the mean rotation angle of the neighbourhood of a point in the deformation field

	Uniaxial tensile	Pure shear	Slide	Simple shear
Transformation function	$x = X$ $y = (1+\lambda)Y$	$x = X + Y\tan\gamma$ $y = X\tan\gamma + Y$	$x = X + Y\sin\gamma$ $y = Y\cos\gamma$	$x = X + Y\tan\gamma$ $y = Y$
$\tan\theta = \dfrac{\mathrm{d}y}{\mathrm{d}x}$	$0(1+\lambda)\tan\Theta$	$\dfrac{\tan\Theta + \tan\gamma}{1 + \tan\Theta\tan\gamma}$	$\dfrac{\tan\Theta\cos\gamma}{1 + \tan\Theta\sin\gamma}$	$\dfrac{\tan\Theta}{1 + \tan\Theta\tan\gamma}$
$\dfrac{\mathrm{d}\theta}{\mathrm{d}\Theta}$	$\dfrac{1+\lambda}{\cos^2\Theta + (1+\lambda)^2\sin^2\Theta}$	$\dfrac{1 - \tan^2\gamma}{1 + \tan^2\gamma + 2\sin 2\Theta\tan\gamma}$	$\dfrac{\cos\gamma}{1 + \sin^2\Theta\sin\gamma}$	$\dfrac{1 + \tan^2\Theta}{(1 + \tan\Theta\tan\gamma)^2 + \tan^2\Theta}$
Θ_1	$\dfrac{\pi}{2}$	$\dfrac{\pi}{4}$	$\dfrac{\pi}{4}$	$\dfrac{\pi}{2} - \dfrac{1}{2}\arctan(2\cot\gamma)$
Θ_2	0	$\dfrac{3\pi}{4}$	$\dfrac{3\pi}{4}$	$-\dfrac{1}{2}\arctan(2\cot\gamma)$
θ_1	$\dfrac{\pi}{2}$	$\dfrac{\pi}{4}$	$\arctan(\sec\gamma - \tan\gamma)$	$-\Theta_2$
θ_2	0	$\dfrac{3\pi}{4}$	$-\arctan(\sec\gamma + \tan\gamma)$	$-\Theta_1$
$\left(\dfrac{\lambda_1}{\lambda_2}\right)^{1/2} = \dfrac{\mathrm{d}\theta(\Theta_2)}{\mathrm{d}\Theta}$	$1+\lambda$	$\dfrac{1+\tan\gamma}{1-\tan\gamma}$	$\sec\gamma + \tan\gamma$	$\left(\dfrac{\sqrt{\tan^2\gamma + 4} + \tan\gamma}{2}\right)^2$
$\bar{\phi}$	0	0	$\dfrac{\gamma}{2}$	$\arctan(\frac{1}{2}\tan\gamma)$

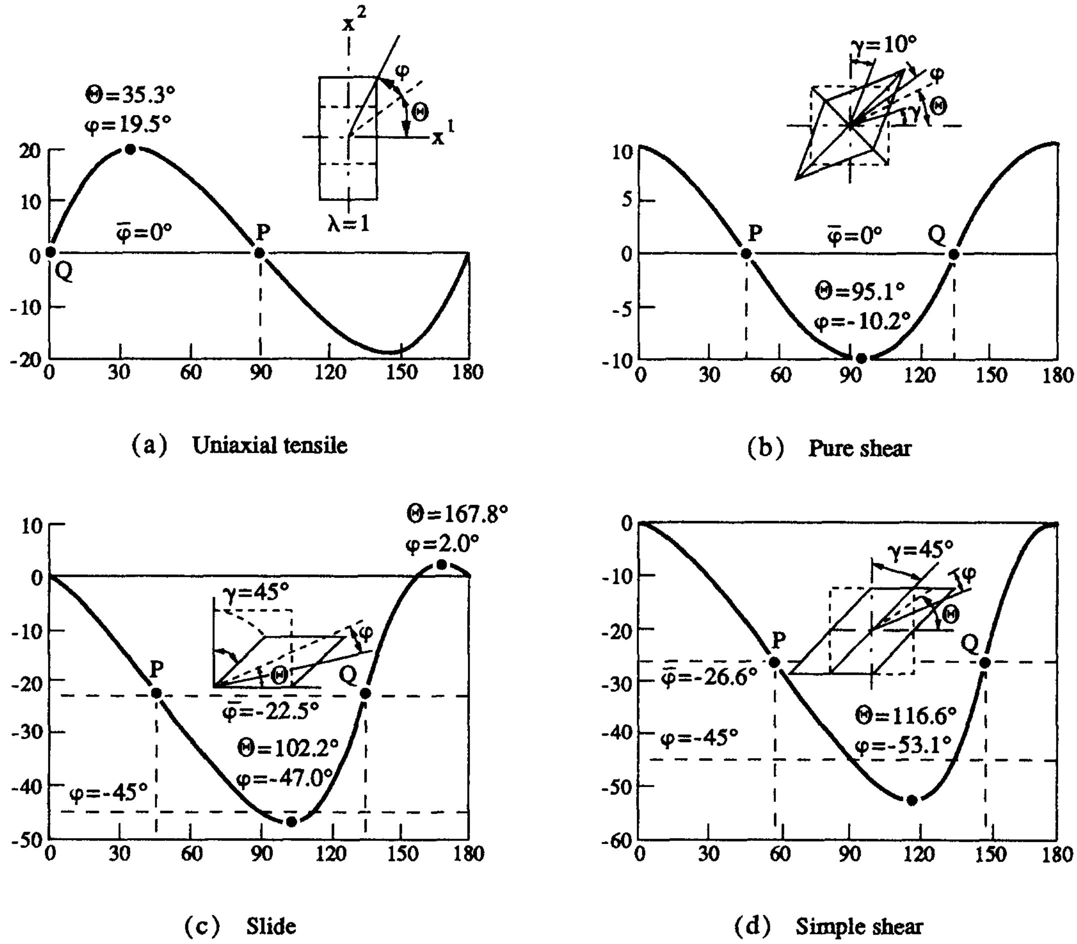

Fig. 2. The curve of φ against Θ shows the distribution of rotation angles of line elements passing through a point. P and Q are the inflection points of the curve and determine the first and second principal directions of strain in the deformation field. $\bar{\varphi}$ is the mean value of rotation angles of all line elements passing through the point.

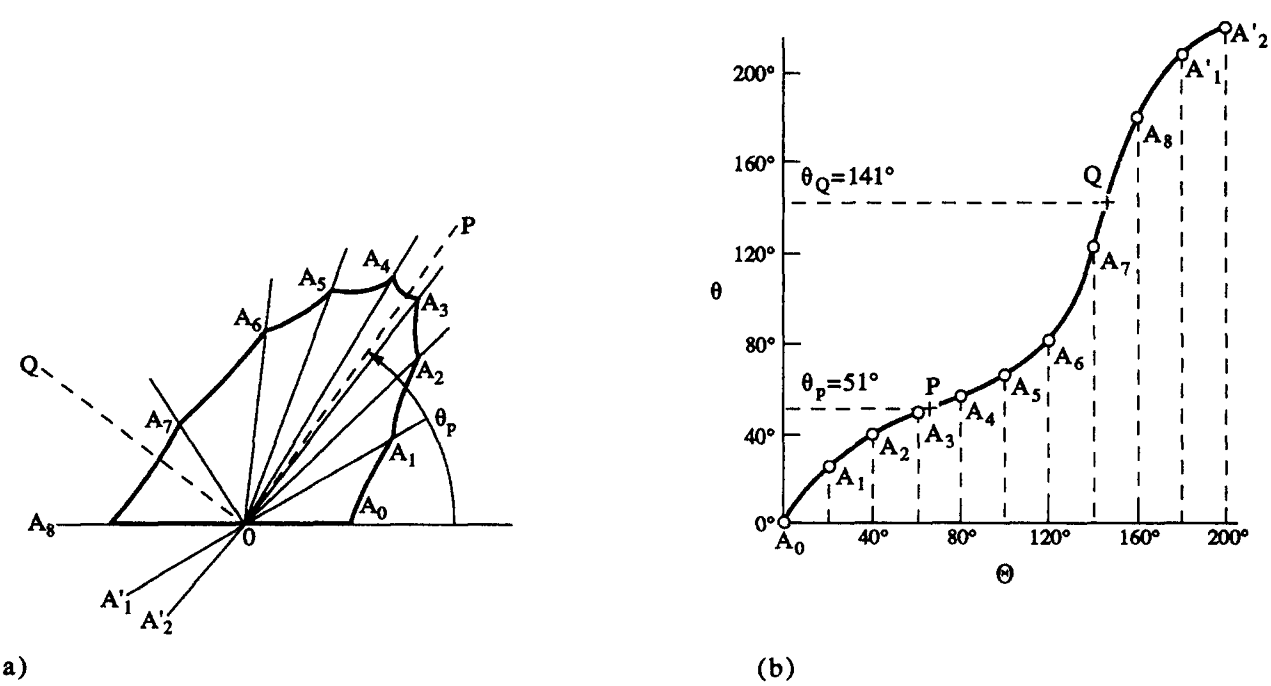

Fig. 3. Application of the extremum rule of angular variation to determine principal directions of strain in a brachiopod fossil. (a) The mark lines OA_0–OA_8 on the surface of the fossil are supposed to scatter regularly in the original fossil. OA_1' and OA_2', respectively, are the extensions of A_1O and A_2O. The dash lines OP and OQ have been determined to be the first and second principle stretch directions by θ_P and θ_Q. (b) The curve of θ against Θ. The initial values of Θ and θ are both defined to OA_0. The angles θ_P and θ_Q, responding to inflection points P and Q of the curve, show the first and second principal directions.

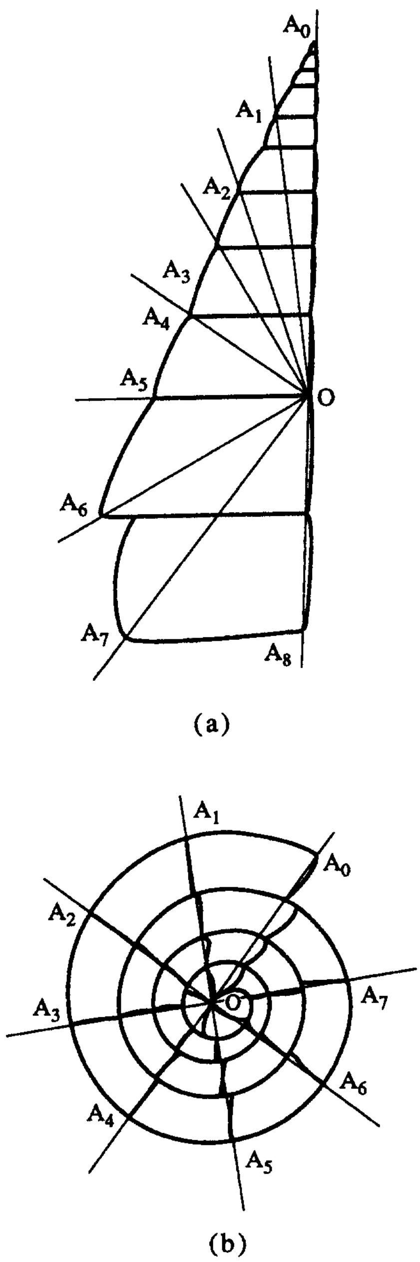

Fig. 4. The imagined radial lines OA_0–OA_8 on the surface of a fossil (a) cephalopod or (b) ammonoids would be used for the principal axes of strain in the fossil, provided an original angle included between one of the lines and OA_0 might be defined by the biological rule of the original body (after Ramsay 1967).

undeformed fossil. For convenience, we define the initial values of both Θ and θ to the same azimuth as OA_0. In accordance with the extremum rule, the angles θ_P and θ_Q, which respond to the inflection points P and Q of the curve (Fig. 3b), give, respectively, the directions of maximum and minimum stretches in the deformed fossil, and the ratio of principal elongation $(\lambda_2/\lambda_1)^{1/2}$ should be equal to the slope of the curve at point P, i.e.

$$(\lambda_2/\lambda_1)^{1/2} = \mathrm{d}\theta(\mathrm{P})/\mathrm{d}\Theta \approx 0.35. \tag{17}$$

It is evident that the accuracy of the calculation of principal stretch axes depends directly upon the number of mark lines on the surface of a deformed fossil and few mark lines bring about considerable error. When a fossil

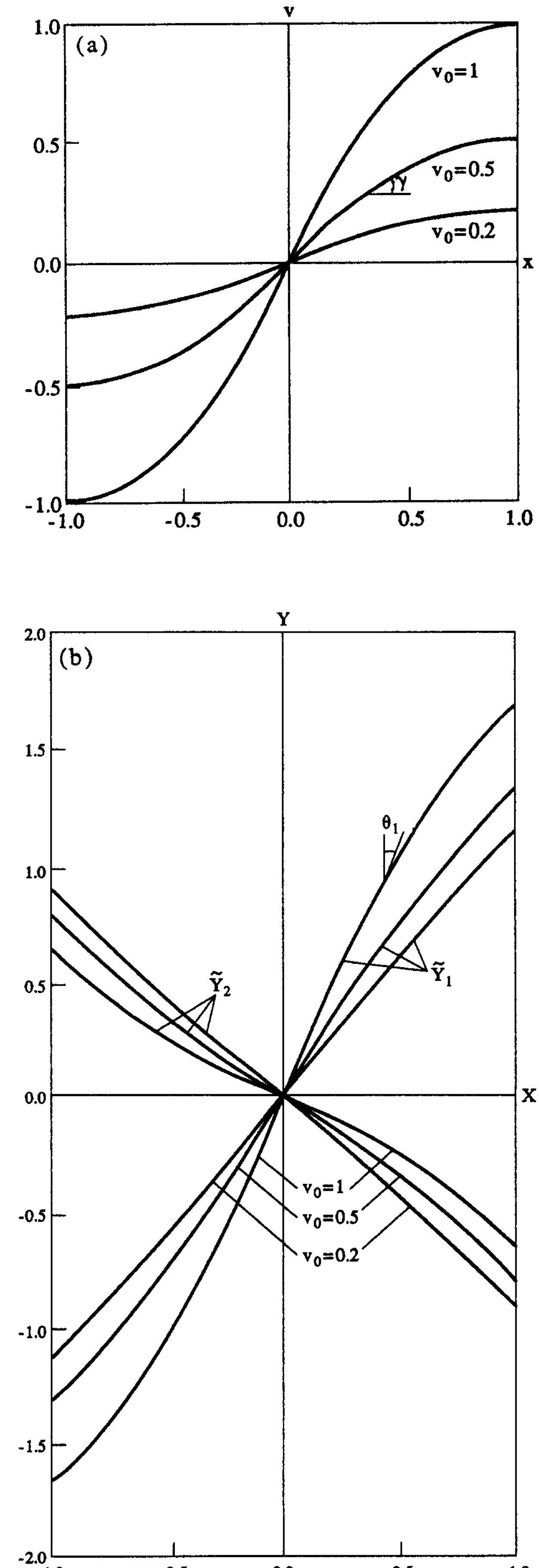

Fig. 5. (a) The distribution of displacement in a heterogeneous shear zone: $v = v_0(2 - |X|)X$. (b) The trajectories of maximal stretch $\tilde{Y}_1(X)$ and minimal stretch $\tilde{Y}_2(X)$.

is deficient in radial mark lines, we may utilize the biological features of the fossil to form new radial lines (Fig. 4), provided the angle included between any two of the new radial lines would be known, both before and after deformation.

EXTREMAL STRETCH TRAJECTORIES IN HETEROGENEOUS SHEAR ZONE

The trajectories of principal stretches in a heterogeneous shear zone is a subject of structural geology (Ramsay 1967, 1980, Ramsay & Graham 1970, Cobbold 1979, 1980, Cobbold & Barbotin 1988). Although it is still difficult to obtain the analytical expressions of the trajectories, the extremum rule of angular variation makes it possible. Here we give an example.

In consideration of the antisymmetry of the zone, we may conduct the study in a half of the zone. Let the width of the zone be two units. $\tilde{Y}_1(X)$ and $\tilde{Y}_2(X)$ express the equations of maximum and minimum stretch trajectories in the deformed state and, $Y_1(X)$ and $Y_2(X)$ in the undeformed state. We assume the displacement function to be (Fig. 5a)

$$v = v_0(2 - X)X, \tag{18}$$

where v_0 is a constant. Shear γ is then determined by

$$\tan\gamma = \mathrm{d}v/\mathrm{d}X = 2v_0(1 - X). \tag{19}$$

From Fig. 5(b), we can set up the following relation

$$\mathrm{d}X/\mathrm{d}\tilde{Y}_1 = \tan\theta_1. \tag{20}$$

But from Table 1, we have

$$\theta_1 = -\Theta_2 = \tfrac{1}{2}\arctan(2\cot\gamma), \tag{21}$$

thus equation (20) becomes

$$\mathrm{d}X/\mathrm{d}\tilde{Y}_1 = 2/(\sqrt{\tan^2\gamma + 4} + \tan\gamma). \tag{22}$$

Solving differential equation (22) with equation (19), we obtain

$$\tilde{Y}_1(X) = \int v_0[\sqrt{(1 - X)^2 + 1/v_0^2} + 1 - X]\mathrm{d}X \tag{23}$$

or

$$\tilde{Y}_1(X) = v_0(1 - X/2)X + \frac{1}{2}v_0(X - 1)\sqrt{(1 - X)^2 + 1/v_0^2} + \frac{1}{2v_0}\ln[X - 1 + \sqrt{(1 - X)^2 + 1/v_0^2}] + C. \tag{24}$$

Using the condition $\tilde{Y}_1(0) = 0$, the constant C is determined as

$$C = \frac{1}{2}v_0\sqrt{1 + 1/v_0^2} - \frac{1}{2v_0}\ln\left(\sqrt{1 + 1/v_0^2} - 1\right). \tag{25}$$

For the whole shear zone, the equation of maximum stretch trajectory is written as

$$\tilde{Y}_1(X) = \mathrm{sign}(X)\Big\{v_0(1 - |X|/2)|X| + \frac{1}{2}v_0(|X| - 1)\sqrt{(1 - |X|)^2 + 1/v_0^2} + \frac{1}{2v_0}\ln[|X| - 1 + \sqrt{(1 - |X|)^2 + 1/v_0^2}] + C\Big\}. \tag{26}$$

Similarly we have

$$\tilde{Y}_2(X) = \mathrm{sign}(X)\Big\{v_0(1 - |X|/2)|X| - \frac{1}{2}v_0(|X| - 1)\sqrt{(1 - |X|)^2 + 1/v_0^2} - \frac{1}{2v_0}\ln[|X| - 1 + \sqrt{(1 - |X|) + 1/v_0^2}] - C\Big\} \tag{27}$$

$$Y_1(X) = -\tilde{Y}_2(X), \qquad Y_2(X) = -\tilde{Y}_1(X). \tag{28}$$

In fact $Y_1(X)$ and $Y_2(X)$, respectively, are the mirror images of $\tilde{Y}_2(X)$ and $\tilde{Y}_1(X)$, if a plane mirror is set along X axis. Figure 5(b) shows the trajectories $\tilde{Y}_1(X)$ and $\tilde{Y}_2(X)$.

In conclusion, the extremum rule of angular variation is an essential rule of finite deformation. It shows the importance of line element rotations in the neighbourhood of a point. As a practical approach to analysis of strain, the extremum rule has wide potential for application in analysis of structural deformation. Some applications will be reported in later papers.

Acknowledgements—The author wishes to express his sincere thanks to Professors P. R. Cobbold and D. De Paor for their comments and helpful suggestions which have considerably improved this manuscript.

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